Non-linearity

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My opinion of how some common non-linear functions relate to each other.
Dirac delta function

$$\delta(x) = \begin{cases} \infty & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases} \tag{1}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1 \tag{2}$$

Integral of Dirac delta function is a step function

$$\int_{-\infty}^{\infty} \delta(x)\, dx = \int_{-\infty}^{0^-} \delta(x)\, dx + \int_{0^+}^{\infty} \delta(x)\, dx \tag{3}$$

$$= \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases} \tag{4}$$

$$= \Delta(x) \tag{5}$$

Integral of step function is a rectified linear unit

$$\int_{-\infty}^{\infty} \Delta(x)\, dx = \int_{-\infty}^{0^-} \Delta(x)\, dx + \int_{0^+}^{\infty} \Delta(x)\, dx \tag{6}$$

$$= \int_{-\infty}^{0^-} 0\, dx + \int_{0^+}^{\infty} 1\, dx \tag{7}$$

$$= \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \tag{8}$$

$$= max(0, x) \tag{9}$$

Resources

1. Ian Goodfellow, Yoshua Bengio and Aaron Courville. Deep Learning book.
   https://www.deeplearningbook.org

2. Adam Taylor. Distributions: What exactly is the Dirac Delta "Function"?
   https://www.cantorsparadise.com/
   distributions-what-exactly-is-the-dirac-delta-function-e2af19d6e700